HALF AND QUARTER WAVELENGTH LINES

In the last section, we saw that RF is reflected from the end of a transmission line that is not terminated in its characteristic impedance. As a consequence, the impedance measured at the input of the transmission may or may not be the same as the load impedance. Before examining the general case of a transmission of an arbitrary electrical length, we will look at two special cases:

Half wavelength lines
Quarter wavelength lines.

HALF WAVELENGTH LINE ($\lambda/2$ LINE)

A half wavelength transmission line is one whose electrical length is one half wavelength long, or a multiple of one half wavelength. When a $\lambda/2$ line whose characteristic impedance is Z_0 is terminated in a load impedance Z_L , the input impedance is always Z_L , regardless of the value of Z_L . To see why this is so, consider an RF sine wave of frequency f and wavelength λ traveling from input to the load. If $V_0\cos(2\pi[(x/\lambda)+ft])$ and $I_0\cos(2\pi[(x/\lambda)+ft])$ are the voltage and current on the transmission line, then at the input, x = 0, and the input impedance is:

$$Z_S = V_0\cos(2\pi ft) / I_0\cos(2\pi ft)$$

At the load, $x = \lambda/2$, and the load impedance of the line is:

$$Z_L = V_0\cos(\pi+2\pi ft) / I_0\cos(\pi+2\pi ft)$$

From elementary trigonometry, we know that $\cos(\pi+2\pi ft) = \cos(2\pi ft)$. Substituting this into the above expression gives:

$$Z_L = V_0\cos(2\pi ft) / I_0\cos(2\pi ft) = Z_S$$

Notice that this is true for any value of Z_0 , Z_S and Z_L.

This property of half wave lines has a very useful consequence

A transmission line may be reduced or increased in length by a half wavelength (or a multiple) and the input impedance remains the same.

Here is an example showing how a half wavelength line could be used:

An FM broadcast antenna has an impedance of 50 ohms. The transmitter requires a 50 ohm load. The only transmission line available is low loss 75 ohm coaxial cable (type RG-6), whose velocity factor is 0.75. If the antenna is located 400 feet above the transmitter and the operating frequency is 89.1 MHz, what is the minimum length of 75 ohm cable required that will connect the transmitter and antenna and provide a 50 ohm input impedance?

Solution:

Since the transmission line has a characteristic impedance of 75 ohms and the load impedance is 50 ohms, the only way to get a 50 ohm input impedance is to select a length of line that is a multiple of a half wavelength long. In fact, we need to determine the multiple of a half wavelength at 89.1 MHz that is closest to and larger than the required line length of 400 ft.

We begin by computing the electrical length of 400 feet of RG-6 coaxial transmission line:

$$l_{ELECTRICAL} = \frac{lf}{984V_F} = \frac{200*98.1}{984*0.75} = \frac{19620}{738} = 26.59\lambda$$

It is necessary to make the line 0.41λ longer, resulting in an overall length of 27λ, which is a multiple of 0.5λ. Note that we cannot take 0.19λoff the line to get 26.5λ, because the line would then be too short. Now we determine the physical length of line equivalent to 0.41λ .

$$\frac{984V_F l_{ELECTRICAL}}{f} = l = \frac{984*0.75*.41}{98.1} = \frac{302.6}{98.1} = 3.084\,ft = 3\,ft\,1in.$$

The required length of line is 400 + 3.084 = 403.084 ft = 403 ft 1 in

A half wave line can also be used in place of a resonant LC circuit. The input impedance of an open circuited half wave line is infinite. If RF of a slightly lower frequency is applied, the electrical length of the line decreases below a half wavelength and the input impedance is inductive. If the frequency is increased, the input impedance is capacitive. Thus the open circuited half wave line acts like a parallel LC circuit. By similar reasoning, one can show that a shorted half wave line acts like a series resonant circuit. The table below shows the relationship between a half wave line and a resonant LC circuit.

Half Wave Line Termination	$f < f_0$	$f = f_0$	$f > f_0$	Type of LC Circuit
open circuit	Z_S is inductive $L_{ELECT} < \lambda/2$	Z_S is infinite $L_{ELECT} = \lambda/2$	Z_S is capacitive $L_{ELECT} > \lambda/2$	Parallel LC
short circuit	Z_S is capacitive $L_{ELECT} < \lambda/2$	Z_S is zero $L_{ELECT} = \lambda/2$	Z_S is inductive $L_{ELECT} > \lambda/2$	Series LC

At very high frequencies, where it is difficult to construct LC circuits with capacitors and inductors, they may be constructed from lengths of transmission line.

QUARTER WAVELENGTH LINE (λ/4 LINE)

A quarter wavelength transmission line is one whose electrical length is one quarter wavelength long, or an odd multiple of a quarter wavelength long. The input, load and characteristic impedances of a quarter wave line are related by the following equation:

$Z_S = Z_0^2 / Z_L$

where:

Z_S is the input impedance
Z_0 is the characteristic impedance of the transmission line
Z_L is the load impedance

Quarter wave lines are generally used to transform an impedance from one value to another. Here is an example:

A VHF loop antenna used to receive weather maps from satellites has an impedance of 110 ohms at 137 MHz. This antenna will be used with a receiver whose input impedance is 50 ohms.

a) What is the impedance of the quarter wave matching section?

b) If a cable with a 0.75 velocity factor is used for the matching section, what is its length?

a) We begin with $Z_S = Z_0^2 / Z_L$. This equation can be rewritten as:

$Z_0^2 = Z_L Z_S$

$Z_0^2 = 110*50 = 5500$

$Z_0 = \text{sqrt}(5500) = 74$ ohms

There are several commercially available transmission lines with characteristic impedances of either 72 ohms and 75 ohms. Any of them would be acceptable in this situation.

b) To determine the actual length we use the following formula:

(984*VF*0.25)/f = length

where VF = the velocity factor and f is the frequency in MHz. If we do the math, we discover that the matching section is 15 inches long.

A quarter wave line can also be used in place of a resonant LC circuit. The input impedance of an open circuited quarter wave line is zero (short circuit). If RF of a slightly lower frequency is applied, the electrical length of the line decreases below a half wavelength and the input impedance is capacitive. If the frequency is increased, the input impedance is inductive. Thus the open circuited quarter wave line acts like a series LC circuit. By similar reasoning, one can show that a open circuited quarter wave line acts like a parallel resonant circuit. The table below shows the relationship between a quarter wave line and a resonant LC circuit.

Quarter Wave Line Termination	$f < f_0$	$f = f_0$	$f > f_0$	Type of LC Circuit
open circuit	Z_S is capacitive $L_{ELECT} < \lambda/2$	Z_S is zero $L_{ELECT} = \lambda/2$	Z_S is inductive $L_{ELECT} > \lambda/2$	Series LC
short circuit	Z_S is inductive $L_{ELECT} < \lambda/2$	Z_S is infinite $L_{ELECT} = \lambda/2$	Z_S is capacitive $L_{ELECT} > \lambda/2$	Parallel LC

In the next section, we will look at the relationship between load and input impedance for a line of arbitrary length and introduce a useful graphical tool that is used to solve transmission line problems.